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# A Biomathematical Model of a Human Operator's Falling Asleep

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**Abstract**—Quantitative analysis of the transition from wakefulness to sleep and prediction of the moment when errors in professional activity appear because of a decrease in the arousal level require microinterval monitoring of falling asleep. A psychomotor test was developed that rapidly decreased the arousal level, which made it possible to record as many as 10–20 episodes of correct and erroneous activity within 40 min and isolate the periods electrophysiologically corresponding to wakefulness and brief sleep. Seventy subjects were tested, and 6700 fragments of recordings with correct and erroneous performance were analyzed. Analysis of the experimental data showed that the transition from wakefulness to sleep includes intermediate short and relatively long periods of wakefulness and sleep, whose durations are distributed according to the double exponential law. A mathematical model describing the time course of alternation of these four states of wakefulness and sleep predicts the probability of prolonged, potentially dangerous disturbances in operator activity because of microsleep as dependent on the initial state and individual characteristics of subjects. The results will be useful both for the development of devices monitoring and predicting changes in the physiological arousal level and for analysis of traffic and industrial accidents.

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## INTRODUCTION

A critical decrease in the arousal level is one of the most common causes of errors in occupational activity [1–3]. Most researchers believe that drowsiness accompanied by short (3- to 10-s) episodes of "microsleep" is a direct cause of various traffic and industrial accidents [3].

The notion on cyclic transition from wakefulness to sleep [3–10], which occupies the entire first stage of sleep and ends no earlier than the second stage of sleep, according to the generally accepted classification [11], is important for understanding the mechanisms of operator errors caused by a decrease in the arousal level. Electrophysiological analysis of the cycles of falling asleep yielded quantitative estimates of the durations of intervals with alternating dominance of wakefulness and sleep [4, 5]. In these studies, bursts of $\alpha$ rhythm served as EEG indicators of the activation of the arousal system, and the periods of $\theta$ waves between these bursts reflected a shift towards dominance of sleep processes. The time between consecutive $\alpha$ bursts with $\theta$-rhythm fragments between them was assumed to be the duration of an activation cycle reflecting alternation of wakefulness and sleep dominances during transition to deeper sleep stages. Measurement of the intervals between consecutive $\alpha$ bursts showed that the mean duration of the activation cycles was 15–18 s at the first stage of sleep [4], increased to 30–34 s by the end of this stage, and was 51–60 s at the second stage of sleep [5].

Behavioral measurement of the duration of intervals when operators made errors against the background of a decreased arousal level yielded similar durations of these intervals. Behavioral tests showed that the intervals were 14–20 s [12, 13]. In studies where two different behavioral tests were used [14, 15], the mean duration of intervals containing activity errors was 18 s.

Thus, only the mean durations of intervals containing operator errors and activation cycles during transition to sleep have been statistically estimated in both behavioral studies [12–15] and observations of "spontaneous" falling asleep [4, 5], whereas the pattern of the cyclic transition from wakefulness to sleep has not been studied.

The existing models of activity disturbance based on current concepts of sleepiness [1, 16, 17] fairly well fit long-term changes in the behavioral parameters of operators as dependent on the night sleep pattern and the time of day/night. However, the factors included in these models are insufficient for predicting behavior during short time intervals for individual subjects under real conditions. Authors of some recent studies believe that, in addition to these factors, the current arousal levels of individual subjects and their specific sleepiness parameters should be taken into consideration for more accurate simulation of operator behavior [18, 19].

Earlier [20, 21], we developed a discrete–continuous psychomotor test where a subject performs monotonous, tedious activity with the eyes closed. A rapid decrease in the arousal level and appearance of short "microsleep" episodes result in errors soon after the start. The use of a behavioral criterion permitted a very precise measurement of the temporal characteristic of intervals with correct and incorrect performance of the psychomotor test in the course of a gradual decrease in the arousal level that, according to electrophysiological criteria, correspond to alternation of the dominances of the sleep and wakefulness cerebral systems [22].

The purposes of this study was to construct a mathematical model of the time course of intervals with correct and incorrect performance of the test and to estimate the possibility of quantitative analysis of individual-specific temporal characteristics of the interaction between the wakefulness and sleep systems in operators falling asleep as a result of monotonous psychomotor activity.

## METHODS

The psychomotor test was carried out as follows. Subjects sitting, with their eyes closed, in a comfortable chair in a dimly lit room had to mentally count 1-s intervals while alternately performing to series of acts: (1) counting from one to ten simultaneously pressing a button, (2) counting from one to five not pressing the button, and so on. The button was sensitive to the force of the pressure, and the subjects held it between the right thumb and index finger. To prevent the button from dropping upon the subject's muscle relaxation caused by drowsiness, it was fixed to the subject's hand with adhesive tape. Before the test, a subject was instructed to relax as much as possible and then to perform the acts described above. The monotony of the test caused signs of drowsiness to appear as soon as within the first 1–3 min of the observation. Sleepiness was also provoked by the time of day (the test were performed between 6 and 8 p.m.). The duration of the experiment was 40–50 min.

During the test, we recorded a univocal electroencephalogram (EEG), with the electrode located at position *CZ* (according to the international 10–20 system), and an electooculogram (EOG), with two electrodes used for recording horizontal and vertical eye movements, respectively. Linked electrodes set at the mastoid processes of the temporal bones of the skull served as a reference electrode. A Maclab 8E polygraph (Australia) was used for recording the data. Electrophysiological signals were digitized at a quantization frequency of 100 Hz. The piezoelectric button sensitive to the force of pressure was 15 mm in diameter and 6 mm in width.

A total of 70 subjects aged from 17 to 68 years (42 men and 38 women) participated in the tests. Most of the study (203 tests) was performed in 13 subjects. The remaining subjects participated in a total of 60 tests, each subject participating in only one or two tests. This is too few to calculate statistically significant model parameters for this group of subjects. However, we took into account their results when calculating the parameters of the summary model for a pooled sample of subjects. Each experimental series lasted for 40–50 min, during which time a subject went to sleep and awakened several times.

## RESULTS

*The behavioral criteria of correct/incorrect performance of the test were the following* (Fig. 1). *Correct performance*: the number of impulses in the series 1 (with pressing the button) was ten; the duration of the series 2 (without pressing the button) corresponded to the duration of pressing the button five times (about 5 s ±30%). *Incorrect performance*: a subject pressed the button either less or more than ten times in series 1; the duration of series 2 differed from the duration of pressing the button five times by more than 30%. The durations of the fragments with correct or incorrect performance were determined to an accuracy of 2–3 s.

In addition to the behavioral criterion, we used electrophysiological parameters. The presence of the EEG $\alpha$ rhythm was a necessary condition for identification of correct performance; and inhibition of the $\alpha$- rhythm and appearance of the $\theta/\Delta$ rhythms, for identification of incorrect performance. These electrophysiological criteria based on the classification of drowsiness stages suggested in [22] allowed us to identify the fragments with correct performance as vigilance (*V*) and those with incorrect performance as drowsiness or short, unsound sleep (*S*). Such short episodes of a decreased arousal level accompanied by a few $\theta$ waves are often called "microsleep"; hereinafter, however, we will refer to them as *sleep or S*, bearing in mind that this term is merely conventional here [11].

Figure 1 shows the procedure for determining fragments with correct and incorrect performance of the test on the basis of behavioral parameters. Fragments with correct and incorrect performance can be seen in the mechanogram of pressing the button. Episodes of errors made during different intervals of time are clearly seen in enlarged fragments of the recording shown at the top. Transition to longer episodes of sleep did not occur at once; it was usually preceded by alternation of several shorter episodes of wakefulness and sleep. Recordings with short and long periods of incorrect performance are shown at the top. A diagram of the subject's state during the experiment is shown at the bottom; the state in which the subject correctly performed the test is denoted 0; the state corresponding to incorrect performance, 1. The $0 \longrightarrow 1$ leap in the diagram corresponds to the appearance of errors in test performance (see the mechanogram of pressing the button), and the reverse transition ($0 \longrightarrow 1$) corresponds to restoration of correct performance. In this way, we formalized the cyclic alternation of periods with dominance of the wakefulness and sleep systems. A total of 6700 fragments of recordings with correct and incorrect

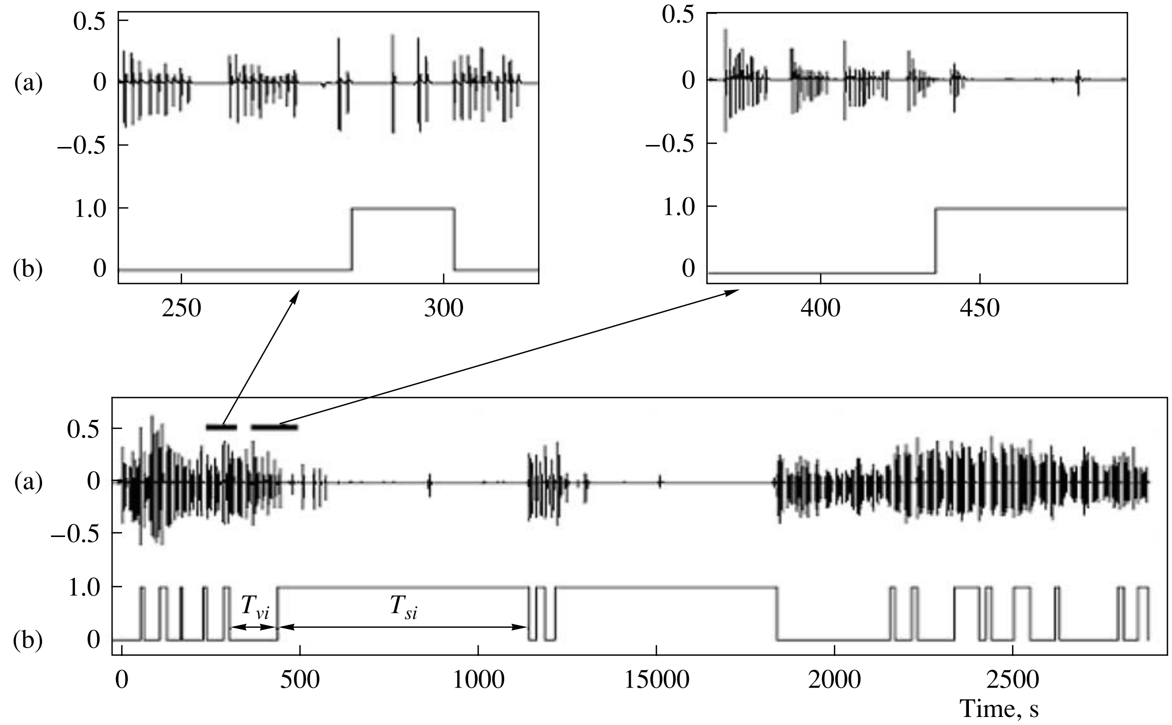


**Fig. 1.** The procedure for identifying the fragments with correct (vigilance) and incorrect (sleep) performance of the psychomotor test (subject 10). (a) A mechanogram of pressing the button (the ordinate shows the force of pressing in arbitrary units); (b) a diagram of the subject's states;) and 1 indicate correct and incorrect performance, respectively. Intervals with correct performance of the test are denoted $T_{vi}$ (*V* stands for vigilance), and those with incorrect performance are denoted $T_{si}$(*S* stands for sleep). Arrows point at insets showing enlarged fragments containing vigilance–sleep transitions of different lengths. The abscissa shows time (seconds).

performance of the test by 70 subjects were analyzed. The durations of the fragments with correct and incorrect performance according to behavioral and electrophysiological criteria were evaluated by expert assessment (double blind method) by two independent specialists and subsequent reconciliation between the results.

*The static model of an operator's falling asleep.* Statistical analysis of experimental distributions of intervals with correct and incorrect performance of the test (vigilance and sleep, respectively) showed that both vigilance and sleep periods could be subdivided into long and brief ones. Figure 2 shows the histograms and distribution density curves for intervals with (a) correct (vigilance; *Tv*) and (b) incorrect (sleep, *Ts*) performance. The experimental function of the distribution density of the intervals (points) was calculated from the histograms of the durations of the (a) vigilance and (b) sleep intervals. The theoretical distribution density curve calculated by the maximum likelihood method [23] fits experimental data well. It is a superposition of two distributions, according to the following equations:

$$f_i(t) = c_{bi}\gamma_{bi}e^{-\gamma_{bi}t} + c_{li}\gamma_{li}e^{-\gamma_{li}t}$$
$$c_{bi} + c_{li} = 1. \qquad (1)$$

The indices *b* and *l* mean brief and long characteristic times of vigilance and sleep periods, respectively. The index *i* assumes one of two values: *v* (vigilance) and *s* (sleep); i.e., we consider two independent distributions of type (1), for vigilance and for sleep. The lower row of the equation means that, at any given moment, the subject is in one of two states, vigilance or sleep.

Thus, static simulation of the experimental results in the form of Eq. (1) allowed us to subdivide vigilance and sleep into four states: long and brief vigilance (LV and BV, respectively) and long and brief sleep (LS and BS, respectively). Table 1 shows the mean intervals for these four states; the variation ranges for each subject (1–13) and all subjects (14) are indicated in seconds.

As can be seen in Fig. 2, the probabilities of the two "brief" states (vigilance, $\boldsymbol{T}_{bv}$ and sleep, $\boldsymbol{T}_{bs}$) were the highest; the subjects were less likely to be in the state of long sleep ($\boldsymbol{T}_{LS}$) or vigilance ($\mathbf{T}_{LV}$). As evident from Table 1, the duration of brief sleep ($\boldsymbol{T}_{BS}$) was the shortest(on average, 14 s), and that of brief vigilance ($\boldsymbol{T}_{BV}$) was the longest (on average, 34 s). Table 2 shows the mean time spent in each state and the range of its variation in the sample of subjects in a more compact form.

Theoretical curves in Fig. 2 fit rather well the experimental distributions. However, testing the likelihood of distribution (1) using the $\chi^2$ test [23] showed that the probability of the hypothesis was significant (from 0.2 to 0.9) only for 5 out of 13 subjects.

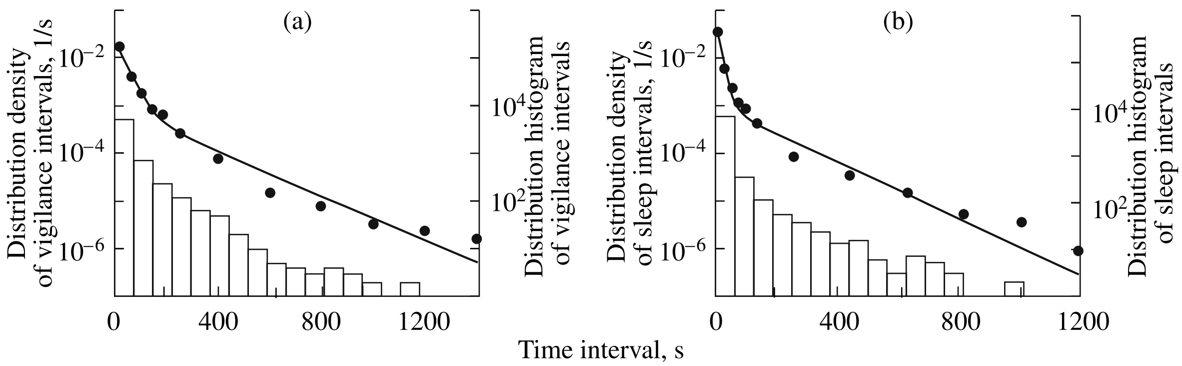


**Fig. 2.** Results of statistical analysis of (a) vigilance intervals ($T_v$) with correct performance of the test and (b) sleep intervals ($T_s$) with incorrect performance. The total number of intervals is 6700. Bar histograms and distribution density curves of the intervals. The right ordinate (for histograms) shows the number of intervals of each length; the left ordinate shows the experimental distribution density of the intervals (points) calculated on the basis of the histogram. The lines are the theoretical distribution density curves, each of which is a superposition of two exponential distributions (see Eq. (1)).

This meant that, first, each subject was characterized by a specific pattern of falling asleep and waking up, these patterns being blurred when parameters for the entire sample of subjects were calculated. Second, even for each particular subject, these patterns varied in different tests. Third, as we will show below, the parameters of distribution (1) changed even within a test.

Let us conventionally call functions (1) of sleep and vigilance distributions a static model, as opposed to a dynamic model of falling asleep, which takes into account changes in a subject's state with time; afterwards, we are going to compare the parameters of the two models.

*The dynamic model of an operator's falling asleep.* These results induced us to turn from a static model of the distribution of sleep and vigilance intervals to description of the dynamics of transition from vigilance to sleep. Analysis of the static model presented above gives enough grounds to hypothesize that falling asleep and waking up in humans is controlled by mechanisms described by exponential laws of distribution, at least two mechanisms being involved in each process. Therefore, the behavior of a subject may be regarded as a system that has four vigilance and sleep states, with random Markovian transitions between them (Fig. 3). The system may be in different states at different moments of time with probabilities complying with Kolmogorov's linear differential equations [24]:

$$\left.\begin{aligned}\frac{dP_1}{dt} &= -(\lambda_1+\lambda_4)P_1+\mu_1P_2+\mu_4P_4;\\ \frac{dP_2}{dt} &= \lambda_1P_1-(\mu_2+\mu_1)P_2+\lambda_2P_3;\\ \frac{dP_3}{dt} &= \mu_2P_2-(\lambda_3+\lambda_2)P_3+\mu_3P_4;\\ \frac{dP_4}{dt} &= \lambda_4P_1+\lambda_3P_3-(\mu_3+\mu_4)P_4.\end{aligned}\right\}, \qquad (2)$$

where the probabilities of the states $P_j(t)$ at any moment of time meet the normalizing relationship

$$P_1+P_2+P_3+P_4 = 1.$$

At the initial moment of time ($t = 0$), the subjects did not sleep; therefore, only the probabilities $P_1(0)$ and

**Table 1.** Mean durations of intervals of four vigilance and sleep states and their variation ranges for each subject (1–13) and all subjects (14) (seconds)

| *NS* | *NT* | Mean duration of intervals of each of the four states | | | |
|---|---|---|---|---|---|
| | | $T_{LV}$ | $T_{BS}$ | $T_{BV}$ | $T_{LS}$ |
| 1 | 6 | **524** | **8** | 108 | **52** |
| 2 | 7 | 205 | 13 | 33 | 216 |
| 3 | 9 | 136 | 18 | 55 | 190 |
| 4 | 10 | 243 | 11 | 49 | 67 |
| 5 | 11 | 153 | 17 | 42 | 128 |
| 6 | 11 | 201 | 10 | 51 | 123 |
| 7 | 13 | 513 | 9 | **135** | **325** |
| 8 | 17 | 148 | 17 | 58 | 114 |
| 9 | 21 | 190 | 10 | 50 | 133 |
| 10 | 23 | 167 | **21** | 62 | 276 |
| 11 | 24 | **73** | 18 | **15** | 119 |
| 12 | 24 | 106 | 14 | 35 | 197 |
| 13 | 27 | 314 | 12 | 27 | 134 |
| 14 | **263** | **86** | **14** | **34** | **147** |

Notes: *NS*, ordinal number of the subject; *NT*, number of the tests performed by the given subject. Parameters: $T_{LV}$, $T_{BS}$, $T_{BV}$, and $T_{LS}$ (see the text), mean durations of intervals. The maximum and minimum values of mean interval lengths in each state for individual subjects and mean interval lengths for all subjects (14) are boldfaced.

**Table 2.** Mean total time in each of the four states and its variation range in the group of subjects

| State | Mean duration, s | Range, s |
|---|---|---|
| *LV* | 193 | 101–485 |
| *BS* | 14 | 9–21 |
| *BV* | 34 | 15–132 |
| *LS* | 146 | 57–269 |

$P_3(0)$ differ from zero; therefore, the initial conditions are the following:

$$P_1(0) = P_{10}, \quad P_2(0) = 0, \quad P_3(0) = 1 - P_{10}, \quad P_4(0) = 0. \quad (3)$$

Equations (2) with initial conditions (3) are the mathematical presentation of the model of an operator's sleep/vigilance. This model describes the change of operator states with time and permits the calculation of the probability of each state at a given moment of time.

The equations of the model contain nine parameters $\lambda_j$, $\mu_j$, and $P_{10}$, which we had to statistically estimate on the basis of experimental data. Simulation of the transitions between these four states required special mathematical tools to be developed using the maximum likelihood and Monte Carlo methods [23]. Their detailed description is beyond the scope of this journal; therefore, we only present the results of the main stages of simulation that are necessary for understanding the physiological basis of the processes described by the model.

Let us consider the solutions of Eqs. (2), which are interesting from both theoretical and practical points of view. They are shown in Table 3 and, in a graphic form, in Fig. 4.

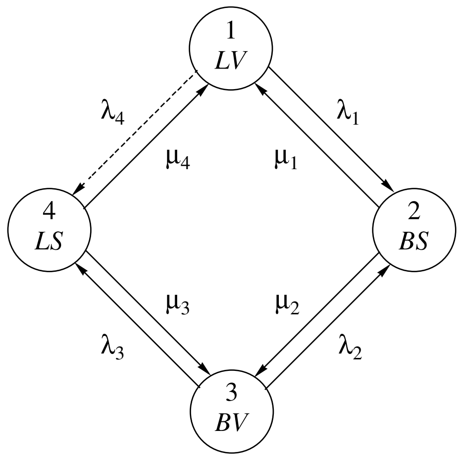


**Fig. 3.** A graph of four vigilance and sleep states of the subjects in the course of falling asleep: (1) long vigilance (*LV*); (2) brief sleep (*BS*); (3) brief vigilance (*BV*); (4) long sleep (*LS*). The system may be in each of the four states with the probability $P_j$. The rates of transitions between the states are designated by the letters $\lambda_j$ and $\mu_j$ at the arrows that show the directions of the transitions: $\lambda_j$– are the rates of falling asleep and $\mu_j$ are the rates of waking up.

The $P_1$ and $P_3$ plots in Fig. 4 show the probabilities that the subject is in the state of long or brief vigilance. Almost all subjects (except for subject 1) were in the state of long vigilance at the initial moment of time: the values on all curves (except that for subject 1) in the $P_1$ plot at the initial moment are 1. The different $P_1$ value for subject 1 indicates that this subject was already sleepy when the experiment began.

The $P_2$ and $P_4$ plots in Fig. 4 show the probabilities that the subject was in the state of brief or long sleep. In both plots, the initial values on all curves (except the curve for subject 1) are zero, which means a zero probability of the state of sleep at the beginning of the test. However, this probability for the "sleepy" subject 1 differed from zero, although it was very small.

Let us use the example of subject 10 to consider how the probability of a dangerously long sleep can be predicted from the results of dynamic simulation. As can be seen in the $P_1$ plot, the probability that long vigilance would occur decreased to zero within less than 10 min (500 s) after the beginning of the test, although the probability of brief vigilance (the $P_3$ plot) reached 0.46. This means that the arousal level of this subject was always decreased within, on average, 10 min after the beginning of the test. As evident from the $P_1$, $P_2$, $P_3$, and $P_4$ plots, this subject, after 10 min of the experiment, could be either in the state of brief vigilance with a probability of about 0.46 (the $P_2$ plot) or in one of the states of sleep: in the state of long sleep with a probability of about 0.42 (the $P_4$ plot) or, with a lower probability (about 0.12) in the state of brief sleep (the $P_2$) plot. As one can see in Table 1, the mean duration of long vigilance ($T_{lv}$) in subject 10 was 0.167 s, the duration of brief vigilance ($T_{bv}$) was 62 s, that of long sleep ($T_{ls}$) was 276 s, and that of brief sleep ($T_{bs}$ = 21 s) was the maximum. Figure 1 shows the results of one test performed by subject 10. As can be seen in the figure, a long sleep episode first appeared at the 430th second, before which brief sleep alternated with brief vigilance. Immediately after the end of the second long sleep episode, at the 1800th second, an episode of long vigilance (almost 300 s) began. As evident from Fig. 1, vigilance intervals were sometimes rather brief even in the beginning of the experiment. Examination of averaged data on subject 10 in the $P_2$ plot (Fig. 4) shows that the curve of the probability of a brief sleep episode peaked (0.15) about 150 s; this means that, in other experiments, brief sleep episodes more often appeared earlier than it is shown in Fig. 1.

The histogram of the time of the first error (Fig. 5) yields additional information on the time course of falling asleep in all subjects. The distribution maximum is about 1–3 min after the start of the test, which means that most subjects made the first error within thin interval of time. The time of the first error is technically determined by the rate of the decrease in the arousal

**Table 3.** Parameters of the dynamic model (2) calculated from experimental data

| *NS* | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $P_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0021 | 0.007 | 0.0015 | 0 | 0 | 0.11 | 0.011 | 0.0053 | 0.52 |
| 2 | 0.0074 | 0.028 | 0.0083 | 0 | 0.026 | 0.058 | 0.0044 | 0 | 1 |
| 3 | 0.0035 | 0.014 | 0.0028 | 0.00031 | 0 | 0.056 | 0.0043 | 0 | 1 |
| 4 | 0.0052 | 0.017 | 0.007 | 0 | 0.02 | 0.075 | 0.017 | 0 | 1 |
| 5 | 0.0043 | 0.02 | 0.0033 | 0 | 0.0018 | 0.059 | 0.0068 | $5 \times 10^{-5}$ | 1 |
| 6 | 0.0045 | 0.015 | 0.0033 | 0 | 0.022 | 0.073 | 0.0071 | 0 | 1 |
| 7 | 0.0021 | 0.0068 | 0.00079 | 0.00018 | 0.0074 | 0.1 | 0.0037 | 0 | 1 |
| 8 | 0.0039 | 0.014 | 0.0019 | 0 | 0 | 0.06 | 0.0078 | 0.0016 | 1 |
| 9 | 0.0095 | 0.015 | 0.01 | 0 | 0.045 | 0.052 | 0.0072 | 0.00028 | 0.82 |
| 10 | 0.0086 | 0.013 | 0.0037 | 0.0002 | 0 | 0.054 | 0.004 | 0 | 1 |
| 11 | 0.0099 | 0.057 | 0.0086 | 0 | 0.001 | 0.056 | 0.0076 | 0 | 1 |
| 12 | 0.0089 | 0.02 | 0.0046 | 0 | 0.0077 | 0.067 | 0.0049 | 0.00026 | 1 |
| 13 | 0.0029 | 0.021 | 0.0043 | 0 | 0.015 | 0.053 | 0.0074 | 0 | 1 |
| 14 | 0.0049 | 0.025 | 0.0051 | 0.00023 | 0.0087 | 0.065 | 0.0061 | 0.00077 | 1 |

Notes: *NS*, ordinal number of the subject. Row 14 shows the summary parameters calculated for all experiments and all subjects. $\lambda_j$, vigilance–sleep transition rates; $\mu_j$, waking rate (see Fig. 3). The dimensionality of the transition rates is 1/s. $P_{10}$, the probability of a subject to be in state 1 (long vigilance) at the beginning of the experiment. The probability of the alternative state 3 (brief vigilance) is $1 - P_{10}$.

level; however, this parameter may reflect the combined effect of several processes: the initial level of sleepiness, i.e., the subject's need for sleep, individual need for activation, and other specific characteristics of the subject. Therefore, individual dynamic simulation, whose results are shown in Fig. 4, provides a more comprehensive description of the changes in the efficiency of operator activity with a decrease in the arousal level.

Let us compare subject 10 with subject 1, for whom the probability of brief vigilance was nonzero in the very beginning of the experiment (Fig. 4, plot $P_3$). According to this parameter, the state of subject 1 in the beginning of the experiment may be interpreted as sleepier than that of subject 10. However, the probabilities of brief and long sleep states for this subject (Fig. 4, plots $P_2$ and $P_4$) were much lower than those for subject 10. Most of the time, subject 1 was in the states of long and brief vigilance (Fig. 4, plots $P_1$ and $P_3$). Table 1 shows that the mean durations of both vigilance intervals in subject 1 (524 and 108 s) were longer than in subject 10, and the durations of both sleep intervals were considerably shorter (52 and 8 s).

Comparison of the results of dynamic simulation for these two subjects demonstrates that estimation of the degree of sleepiness of a subject before the test is insufficient for individual prediction of dangerous episodes of sleep; it is also necessary to know the individual-specific pattern of interaction between the vigilance and sleep systems in the course of activity under the conditions of a gradually decreasing arousal level.

## DISCUSSION

The main result of our study is quantitative description of individual-specific patterns of the cyclic transition from wakefulness to sleep. Four intermediate states have been distinguished on the basis of behavioral and electrophysiological parameters: brief and relatively long vigilance and sleep states. We propose a mathematical model describing the pattern of alternation of these four states of vigilance and sleep that allows prediction the probability of appearance and the duration of each of these states depending on the initial background state and individual characteristics of the subjects.

At present, there are two main approaches to the analysis of the transition from wakefulness to sleep. The first on is based on distinguishing drowsiness as a separate state and successive stages of deepening the drowsiness. Three to nine stages of drowsiness are distinguished, depending on the parameters used [22, 25]. The other approach is based on the notion of cyclic transition from wakefulness to sleep [3–10], which takes up the entire first stage of sleep and is completed only during the second stage, according to the generally accepted classification [11].

The first approach is more traditional and is based on visual analysis of polysomnographic data routinely used in clinical somnology [11]. This approach implies the existence of a single activating cerebral system, which is gradually inhibited as a subject is falling asleep. Rechtschaffen and Kales' generally accepted classification of sleep stages [11] is based on similar assumptions. This approach is justified for clinical use; however, data obtained during past decades indicate

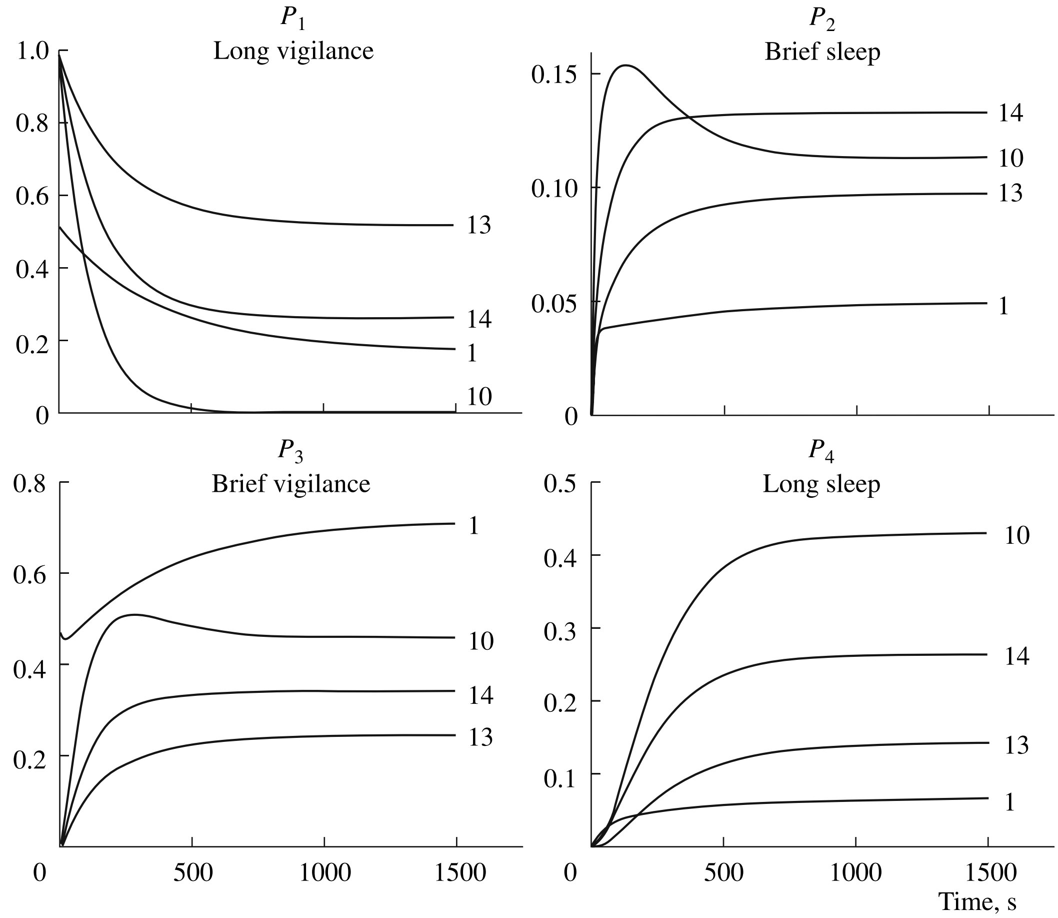


**Fig. 4.** Change in the probabilities $P_i$ ($i = 1, \ldots, 4$), for subjects 1, 10, and 13 to be in each of the four vigilance and sleep states. Curve 14 shows averaged data. The plots have been calculated by Eqs. (2) with the parameters shown in Table 1.

that there are several activating and somnogenic systems in the brain [26].

The model proposed of Hobson et al. [26] is an example of the second approach. This is one of the best known models where the complex interactions between the system controlling wakefulness and sleep is taken into consideration for the analysis of interrelationships between mentality and brain states in the sleep–wakefulness continuum. Hobson et al. carefully analyze how the complex, heterochronous interaction between cerebral structures may determine different ways of the transition from wakefulness to sleep both in the normal state and in various pathologies of sleep.

We have developed our mathematical model of falling asleep in the framework of the second approach. Some authors [26–28] criticize the generally accepted classification of sleep stages according to Rechtschaffen and Kales [11] because it fails to take into account the ample evidence for the mosaic sleep/wakefulness pattern during the night sleep. A dynamic model of interaction between the sleep and wakefulness systems during the night sleep of humans and animals has been proposed [27, 28]. This model is similar to our model presented here, although it employs different mathematical tools. In these studies, only electrophysiological criteria were used for identifying intervals of time where sleep or activation was dominant. These authors used a binary classification not taking into consideration the sleep depth [27, 28].

We used an integrated criterion combining electrophysiological and behavioral parameters to obtain data on the durations of intervals with vigilance or sleep dominance in the course of falling asleep. In contrast to the study of night sleep [27, 28], obtaining of sufficient statistical data on the durations of sleep and vigilance intervals in the course of falling asleep required numerous repetitive experiments with the same subjects. One more methodical aspect that helped us to increase the amount of experimental data was that the psychomotor test was performed in the sitting position, which considerably prolonged the transition to sleep compared to a similar test in the lying position [29].

In our opinion, data on the probability of relatively long episodes of sleep as early as the initial stages of falling asleep (as can be seen in the $P_4$ plot in Fig. 4) is the most interesting result of our study. This result con-

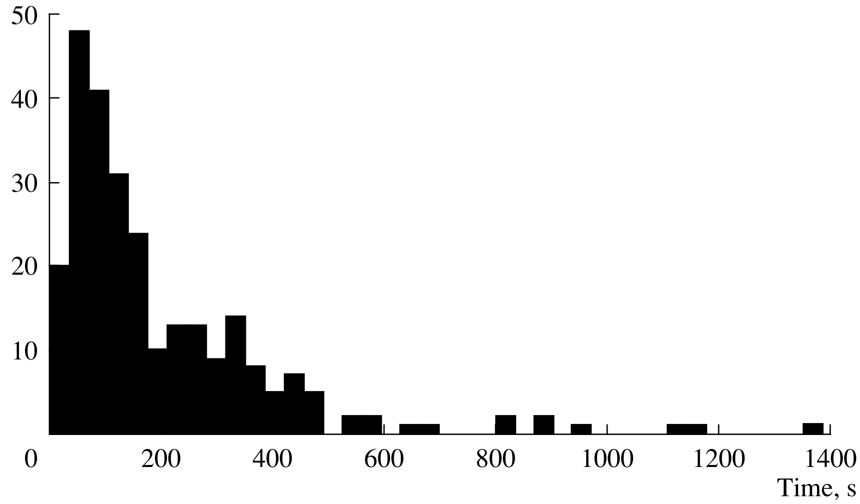


**Fig. 5.** The histogram of the time of the first error for all the 263 tests. The abscissa shows time; the ordinate shows the frequency of the occurrence of the first error in the given interval of time.

tradicts the intuitive notion of gradual transition to sleep and confirms the cyclic, probabilistic nature of this process determined by heterochronous interaction between the wakefulness and sleep systems [26]. This indicates that the question of the moment of sleep onset [10, 11] is incorrect; it would be more correct to speak of the degree of dominance of the sleep system in the course of falling asleep. This is all the more important as increasingly more numerous data are being obtained on the multifunctional nature of different stages of night sleep and the restorative functions of brief day sleep [30]. In this connection, Rechtschaffen and Kales' [11] classification of sleep stages may be regarded as qualitative description of the degree of dominance of different physiological processes in the course of sleep.

From the behavioral viewpoint, if a subject performs a continuous activity involving effector interaction with the environment, sleep begins when the activity stops. This dangerous state often goes unnoticed by the subject, which is the cause of many traffic and industrial accidents [3]. The results of our study may be useful for designers of technological systems for arousal level monitoring. Ideally, these systems should be adjusted individually, with various factors determining the triggering threshold according to the set probability of sleep episodes of a specified duration taken into account [18, 19]. Obviously, the critical temporal characteristics of sleep episodes are different for operators of different transportation vehicles. For example, this time should be much shorter for drivers of cars and trucks, moving at high speeds under variable traffic conditions, than for train drivers, where the road situation is more predictable [31].

## CONCLUSIONS

(1) A psychomotor test has been developed that makes it possible to obtain as many as 20 episodes of erroneous activity caused by a decreased arousal level in a 40- to 50-min experiment. The accuracy of estimation of time intervals with correct and incorrect activities (the vigilance and sleep states) is 1–3 s.

(2) Static simulation of falling asleep has demonstrated that intervals with correct and incorrect activities may be classified into two types, brief and long. The mean duration of the long vigilance is several minutes, and that of the brief vigilance is slightly shorter than 1 min. The brief sleep lasts, on average, about 15 s, which is an order of magnitude shorter than the long sleep.

(3) Analysis of the alternation of these four states of vigilance and sleep has been used to construct a dynamic model of falling asleep, which describes this process in terms of transitions between the two main vigilance and two main sleep states (brief and long). This model permits individual-oriented prediction of the probabilities of correct and erroneous activities of a subject at a given moment of time. The results of this study may be useful for designing devices monitoring the operator's arousal level in traffic and industry.

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